

Calibration algorithm for spatial partial equilibrium models with conjectural variations

Tobias Baltensperger^{a,b,*}, Rudolf M. Füchslin^c, Pius Krütli^a, John Lygeros^b

^a*Institute for Environmental Decisions (IED), ETH Zürich, 8092 Zürich, Switzerland*

^b*Automatic Control Laboratory (IFA), ETH Zürich, 8092 Zürich, Switzerland*

^c*Institute of Applied Mathematics and Physics (IAMP), ZHAW Zürich University of Applied Sciences, 8401 Winterthur, Switzerland*

Abstract

When calibrating spatial partial equilibrium models with conjectural variations, some modelers fit the suppliers' sales to the available data in addition to total consumption and price levels. While this certainly enhances the quality of the calibration, it makes it difficult to accommodate user-imposed bounds on the model parameters such as restricting the market power parameters to the interval $[0,1]$, which is a common requirement in conjectural variations approaches. We propose an algorithm to calibrate the suppliers' sales and simultaneously deal with user-defined bounds on parameters. To this end, we fix the suppliers' sales at reference values and obtain the marginal costs for each supplier and market. We then limit the market power parameters to the interval $[0,1]$, and calculate intervals of anchor prices and price elasticities that reproduce the reference supplier sales in the state of equilibrium. If these intervals do not contain the reference price elasticities and prices, we face a mismatch between reality and the model mechanics. We resolve this by altering the reference sales for the critical suppliers, and iterate. Thereby, the user controls whether price elasticities and anchor prices should be close to their reference values, or the suppliers' sales. The algorithm is tested on data from the European gas market, and required less than one minute to identify calibrated parameters. Our algorithm is widely applicable, since it is based on mild (and common) underlying assumptions and can be configured to suit a specific purpose thanks to the inclusion of user-defined bounds on all relevant parameters.

Keywords: OR in energy, Conjectural variations model, Iterative calibration algorithm, Linear complementarity program

*Corresponding author. Tel.: +41 44 632 44 73;

Email address: t.baltensperger@usys.ethz.ch (Tobias Baltensperger)

1. Introduction

With the advancement in computer technology, an increasing number of market studies is based on mathematical models. A popular way of modeling a network of markets is via a spatial partial equilibrium model including conjectural variations (CV). However, the calibration problem associated with these models turns out to be a Mathematical Program with Equilibrium Constraints (MPEC), as one aims to minimize the deviations between some of the parameters of the model and their reference values subject to the original equilibrium problem. Unfortunately, MPECs have a non-convex solution space and are thus difficult and time-consuming to solve for large models.

An alternative method is used by Lise et al. [18], Cobanli [3] and other gas market modelers. First, consumption per node and time period is fixed and, under the assumption of perfectly competitive markets, the nodal prices are computed. Then the inverse demand functions are derived from predefined price elasticities and the obtained consumption-price pairs, and the equilibrium is computed again under the assumption of imperfectly competitive markets; this reduces consumption and increases price levels. The inverse demand functions are successively shifted outwards until the original consumption levels are matched.

The resulting nodal prices also depend on the predefined market power parameters (conjectural parameters) of the suppliers and the price elasticities of the consumers; some modelers consider these parameters as well to achieve better results. Chyong & Hobbs [2] tune the price elasticities while leaving the market power parameters at their fixed values. While the achieved equilibrium is impressively close to what is observed in reality, this method can yield unrealistically price-inelastic consumers.

In contrast, García-Alcalde et al. [11], Liu et al. [19] and other electricity market modelers [21, 20, 5, 17, 6, 4] tune the market power parameters while leaving price elasticities fixed. The strength of this approach is that the information contained in the level and spatial distribution of the sales of the suppliers is used for calibration, instead of basing the values of the market power parameters on the experience of the modeler. On the downside, the marginal supply costs of each trader have to be known. While this can be approximated by the production costs in the single-market settings of García-Alcalde et al. [11] and the other modelers, the task is more difficult in a network of markets, because transport costs, congestion fees, etc. have to be included. Moreover, the market power parameters are allowed to take arbitrary values, even though, in the absence of cartels, only values between 0 and 1 can be associated with the CV approach, see for instance Tremblay & Tremblay [22, Chapter 12].

In this work, we propose a new calibration algorithm that bridges these gaps by finding market power parameters in the interval $[0, 1]$ based on reference data on sales from individual suppliers to consumers, while maintaining wholesale price levels, price elasticities, and nodal consumption within user-defined bounds.

2. Calibration Framework

2.1. Oligopolistic market representation

We describe an oligopolistic market by an equilibrium problem comprising the Karush-Kuhn-Tucker (KKT) conditions of the optimization problems of the f traders supplying the market, and the market clearing condition.

$$0 \leq -\lambda - \theta_f \frac{d\Lambda(s)}{ds} q_f + \phi_f \perp q_f \geq 0 \quad \forall f \in \mathcal{F} = \{f_1, \dots, f_{\bar{f}}\}. \quad (2.1)$$

$$0 \leq -\lambda + \Lambda(s) \perp \lambda \geq 0 \quad (2.2)$$

λ is the market price, θ_f is the market power parameter of trader f in the set of all traders \mathcal{F} with access to market, $\Lambda(s)$ is the inverse demand function, $s := \sum_{f \in \mathcal{F}} q_f$ is the total consumption in the market, q_f is the quantity sold by trader f , and ϕ_f is trader f 's marginal cost of supplying q_f .

We require the inverse demand function $\Lambda(s)$ to be bijective and have the following characteristics in its anchor point (s^0, λ^0) :

$$\Lambda(s^0) = \lambda^0 \geq 0, \quad \left. \frac{d\Lambda(s)}{ds} \right|_{s^0} = \frac{\lambda^0}{s^0 \cdot \eta} < 0, \quad (2.3)$$

where $s^0 > 0$ is the anchor consumption, $\lambda^0 \geq 0$ the anchor price, and $\eta := \left. \frac{\partial s}{\partial \lambda} \right|_{\lambda^0} \cdot \frac{\lambda^0}{s^0} < 0$ the price elasticity of demand in (s^0, λ^0) . While s^0 , λ^0 , and η are subject to calibration, as discussed in the next section, the functional form of $\Lambda(s)$ can be freely chosen by the modeler.

2.2. Problem statement

When calibrating CV models, the goal is to tune parameters s^0 , λ^0 , η , and θ , such that the resulting model equilibrium $(\lambda^*, q^*, s^*, \phi^*)$ matches reference values $(\lambda^{ref}, q^{ref}, s^{ref} := \sum_{f \in \mathcal{F}} q_f^{ref}, \phi^{ref})$. θ denotes the vector $[\theta_1, \dots, \theta_{\bar{f}}]^T$ or point $(\theta_1, \dots, \theta_{\bar{f}})$, depending on context; q_f and ϕ_f are abbreviated similarly.

In fact, by defining the market power parameters

$$\theta_f(\lambda^0, q_f^{ref}, s^0, \phi_f^{ref}, \eta) := \frac{\lambda^0 - \phi_f^{ref}}{\frac{\lambda^0}{s^0 \cdot (-\eta)} \cdot q_f^{ref}} \quad \forall f \in \mathcal{F}, \quad (2.4)$$

and setting the anchor parameters $s^0 = s^{ref}$, $\lambda^0 = \lambda^{ref}$, and $\eta = \eta^{ref}$, we can force the equilibrium to be in $(\lambda^{ref}, q^{ref}, s^{ref}, \phi^{ref})$ and thereby successfully terminate the calibration. This can be derived by solving Equation (2.2) for s^{ref} , and substituting (2.2) and (2.3) into (2.1); Liu et al. [19] also follow this approach.

Calibrating the model in this way may not be possible due to the following reasons: In contrast to the other parameters and variables, the required reference marginal supply cost ϕ^{ref} is difficult to obtain from data for a network of markets

due to its dependency on q^{ref} and the cost structure of the underlying network; Furthermore, $\theta_f(\lambda^{ref}, q_f^{ref}, s^{ref}, \phi_f^{ref}, \eta^{ref})$ is neither guaranteed to be in the interval $[0, 1]$ for all $f \in \mathcal{F}$, nor properly defined for those traders whose sales $q_f^{ref} = 0$.

In the remainder of this section we derive three modules, based on which we then formulate a calibration algorithm overcoming these difficulties.

- Module I (Section 2.3) introduces how the marginal costs ϕ^{ref} of supplying q^{ref} can be determined using the model.
- Module II (Section 2.4) derives ranges on the anchor price $\lambda^0 \in [\underline{\lambda}^0, \overline{\lambda}^0]$ and the price elasticity $\eta \in [\underline{\eta}, \overline{\eta}]$ for which the market power parameters $\theta_f(\lambda^0, q_f^{ref}, s^{ref}, \phi_f^{ref}, \eta)$ are in the interval $[0, 1]$ for all $f \in \mathcal{F}$.
- If these ranges do not contain λ^{ref} and η^{ref} , we face a basic misalignment between reference data and market equilibria which the model can generate, and thus have to relax the constraints on the reference values. In this case we allow any $\lambda^{ref} \in [\underline{\lambda}^{ref}, \overline{\lambda}^{ref}]$ and $\eta^{ref} \in [\underline{\eta}^{ref}, \overline{\eta}^{ref}]$, and any equilibrium $(\lambda^{ref}, q^{ref}, s^{ref}, \phi^{ref})$ to terminate the calibration, where $\underline{\lambda}^{ref}, \overline{\lambda}^{ref}, \underline{\eta}^{ref}, \overline{\eta}^{ref}$ are user-defined bounds on the reference values, for instance derived from uncertainty inherent to the reference data¹. If the ranges $[\underline{\lambda}^0, \overline{\lambda}^0]$ and $[\underline{\eta}, \overline{\eta}]$ are empty, or do not overlap with $[\underline{\lambda}^{ref}, \overline{\lambda}^{ref}]$ and $[\underline{\eta}^{ref}, \overline{\eta}^{ref}]$, we also allow changes in q^{ref} . Module III (Section 2.5) describes how a new q^{ref} can be found, for which the ranges $[\underline{\lambda}^0, \overline{\lambda}^0]$ and $[\underline{\eta}, \overline{\eta}]$ are shifted towards $[\underline{\lambda}^{ref}, \overline{\lambda}^{ref}]$ and $[\underline{\eta}^{ref}, \overline{\eta}^{ref}]$.

Module I-III are integrated to a calibration algorithm in Section 2.6. Notation is introduced as we proceed, and is summarized in Appendix A.

2.3. Module I: Determining the marginal costs ϕ^{ref} of supplying the reference sales of the traders q^{ref}

We introduce

$$0 = q_f^{ref} - q_f \perp \xi_f \text{ (free)} \quad \forall f \in \mathcal{F}, \quad (2.5)$$

which are additional KKT conditions from adding the constraint $q_f = q_f^{ref}$ to the original optimization problem of each trader f . ξ_f are the shadow prices associated with these constraints and therefore also appear in Equation (2.1), which now reads

$$0 \leq -\lambda - \theta_f \frac{d\Lambda(s)}{ds} q_f + \phi_f + \xi_f \perp q_f \geq 0 \quad \forall f \in \mathcal{F}. \quad (2.6)$$

¹As reference data usually comes with some uncertainty, we would argue that this approach is legitimate. Alternatively, the cost structure of the network, or the model altogether could be changed. In this work, however, we do not investigate those options.

We solve the augmented model comprising Equations (2.2), (2.5), and (2.6), choose parameters $s^0 = s^{ref}$, $\lambda^0 = \lambda^{ref}$, $\eta = \eta^{ref}$, and any $\theta \succeq \mathbf{0}$, where $\mathbf{0}$ is the zero vector of appropriate dimension and \succeq is interpreted component-wise. The variable ξ_f corresponds to a tax/subsidy imposed on the trader f to supply the quantity q_f^{ref} , and compensates for the potential mismatch of supply and demand in the market caused by our (arbitrary) choice of θ . In equilibrium, the vector ϕ reveals the marginal costs of supplying q^{ref} , and we obtain $\phi^{ref} := \phi^*$.

2.4. Module II: Determining admissible ranges for the anchor price λ^0 and the price elasticity η

We obtain all values for the anchor price λ^0 and the price elasticity η for which the market power parameters $\theta_f(\lambda^0, q_f^{ref}, s^{ref}, \phi_f^{ref}, \eta)$ are in the interval $[0, 1]$ for all $f \in \mathcal{F}$ by exploring the dynamics inherent to the model. Note that $\mathcal{F} = \mathcal{F}_+ \cup \mathcal{F}_0 = \{f \in \mathcal{F} | q_f^{ref} > 0\} \cup \{f \in \mathcal{F} | q_f^{ref} = 0\}$.

For $f \in \mathcal{F}_+$, and provided $\theta = \theta(\lambda^0, q^{ref}, s^{ref}, \phi^{ref}, \eta)$, Equation (2.1) implies that the following equation holds in equilibrium.

$$\lambda^0 - \theta_f \frac{\lambda^0}{s^{ref} \cdot (-\eta)} q_f^{ref} = \phi_f^{ref} \quad \forall f \in \mathcal{F}_+ \quad (2.7)$$

Since $\theta_f \frac{\lambda^0}{s^{ref} \cdot (-\eta)} q_f^{ref}$ is non-negative for all valid choices of λ^0 , η , and $\theta_f \in [0, 1]$, we conclude that the wholesale price in equilibrium $\lambda^* = \lambda^0$ is at least as large as the highest marginal supply cost of the supplying traders.

$$\lambda^0 \geq \max_{f \in \mathcal{F}_+} \phi_f^{ref} \quad (2.8)$$

Furthermore, λ^0 is at most equal to marginal cost of the supplying trader with the lowest cost plus its maximum market power markup.

$$\lambda^0 \leq \phi_f^{ref} + 1 \cdot \frac{\lambda^0}{s^{ref} \cdot (-\eta)} q_f^{ref} \quad \forall f \in \mathcal{F}_+. \quad (2.9)$$

We can reformulate condition (2.9) to provide an upper bound on $(-\eta)$ instead of λ^0 :

$$(-\eta) \leq \min_{f \in \mathcal{F}_+} \frac{\lambda^0}{\lambda^0 - \phi_f^{ref}} \cdot \frac{q_f^{ref}}{s^{ref}}. \quad (2.10)$$

For $f \in \mathcal{F}_0$ Equation (2.1) reads

$$\lambda^0 \leq \phi_f^{ref}, \quad (2.11)$$

limiting the price in equilibrium λ^0 to the lowest marginal cost among the non-supplying traders with access to the market:

$$\lambda^0 \leq \min_{f \in \mathcal{F}_0} \phi_f^{ref}. \quad (2.12)$$

Summarizing, if we limit θ to the interval $[0, 1]$ and consider the properties (2.3), λ^0 and η have to be in the ranges

$$\underline{\lambda}^0 := \max_{f \in \mathcal{F}_+} \phi_f^{ref} \leq \lambda^0 \leq \min_{f \in \mathcal{F}_0} \phi_f^{ref} =: \overline{\lambda}^0, \quad (2.13)$$

$$\underline{(-\eta)} := 0 < (-\eta) \leq \min_{f \in \mathcal{F}_+} \frac{\lambda^0}{\lambda^0 - \phi_f^{ref}} \cdot \frac{q_f^{ref}}{s^{ref}} =: \overline{(-\eta)}. \quad (2.14)$$

If the boundary values of the admissible ranges satisfy $\overline{\lambda}^0 \geq \underline{\lambda}^0$ and $\overline{(-\eta)} > \underline{(-\eta)}$, then the equilibrium $(\lambda^*, q^*, s^*, \phi^*) = (\lambda^0, q^{ref}, s^{ref}, \phi^{ref})$ exists, and can be achieved by choosing any λ^0 and η in the ranges $[\underline{\lambda}^0, \overline{\lambda}^0]$, $[\underline{\eta}, \overline{\eta}]$, setting $\theta = \theta(\lambda^0, q_f^{ref}, s^{ref}, \phi_f^{ref}, \eta)$ for all $f \in \mathcal{F}_+$, and setting θ_f to any value in the interval $[0, 1]$ for all $f \in \mathcal{F}_0$; note that in all cases $\theta_f \in [0, 1]$ as desired.

2.5. Module III: Improving the anchor price and price elasticity ranges $[\underline{\lambda}^0, \overline{\lambda}^0]$ and $[\underline{\eta}, \overline{\eta}]$ by altering the reference sales of the traders

If the anchor price and price elasticity ranges $[\underline{\lambda}^0, \overline{\lambda}^0]$ and $[\underline{\eta}, \overline{\eta}]$ do not intersect with $[\underline{\lambda}^{ref}, \overline{\lambda}^{ref}]$ and $[\underline{\eta}^{ref}, \overline{\eta}^{ref}]$, we know that we cannot achieve the equilibrium $(\lambda^{ref}, q^{ref}, s^{ref}, \phi^{ref})$ without violating $\lambda^0 \in [\underline{\lambda}^{ref}, \overline{\lambda}^{ref}]$, $\eta \in [\underline{\eta}^{ref}, \overline{\eta}^{ref}]$, or $\theta_f \in [0, 1]$ for some $f \in \mathcal{F}$. We follow a two-step procedure to find a new equilibrium $(\lambda^{ref}, q^{new}, s^{ref}, \phi^{new})$ for which none of the parameters λ^0 , η , and θ violate their respective intervals.

In a first step, we pick a $\lambda^0 \in [\underline{\lambda}^{ref}, \overline{\lambda}^{ref}]$ and $\eta \in [\underline{\eta}^{ref}, \overline{\eta}^{ref}]$, which are as close as possible to $[\underline{\lambda}^0, \overline{\lambda}^0]$ and $[\underline{\eta}, \overline{\eta}]$, respectively. Furthermore, we fix the anchor consumption s^0 at s^{ref} . We calculate $\theta_f(\lambda^0, q_f^{ref}, s^0, \phi_f^{ref}, \eta) =: \theta_f^{ref}$ for all $f \in \mathcal{F}_+$; these values can be outside the interval $[0, 1]$. Furthermore, we define $\theta_f^{lim} := \min(\max(\theta_f^{ref}, 0), 1)$ for all $f \in \mathcal{F}_+$, and set $\theta_f^{lim} = \sum_{f' \in \mathcal{F}_+} \frac{\theta_{f'}^{lim} \cdot q_{f'}^{ref}}{q_{f'}^{ref}}$ for all $f \in \mathcal{F}_0$, as reference data does not provide any information on the behavior of traders f with $q_f^{ref} = 0$; other approximations could be used as well. From Equation (2.1) we estimate the sales in the new equilibrium:

$$q_f^{est} := \frac{\lambda^0 - \phi_f^{ref}}{\lambda^0} \frac{s^0 \cdot (-\eta)}{\theta_f^{lim}} \quad \forall f \in \{\mathcal{F} | \theta_f^{lim} > 0\}, \quad (2.15a)$$

$$q_f^{est} := q_f^{ref} - \left| \frac{\lambda^0 - \phi_f^{ref}}{\lambda^0} \right| \frac{s^0 \cdot (-\eta)}{1} \quad \forall f \in \{\mathcal{F} | \theta_f^{lim} = 0\}. \quad (2.15b)$$

Note that $q_f^{est} = q_f^{ref}$ if θ_f^{ref} is in the interval $[0, 1]$.

In a second step, we want to find a new equilibrium in the direction of q^{est} . To this end, we augment the original model by

$$0 \leq \max(q_f^{ref}, q_f^{est}) - q_f \perp \overline{\xi}_f^F \geq 0 \quad \forall f, \quad (2.16a)$$

$$0 \leq -\min(q_f^{ref}, q_f^{est}) + q_f \perp \underline{\xi}_f^F \geq 0 \quad \forall f, \quad (2.16b)$$

$$0 = s^0 - s \perp \chi \quad (\text{free}). \quad (2.16c)$$

These complementarity constraints are the KKT conditions of additional constraints in the traders' optimization problems. $\bar{\xi}_f^F$, $\underline{\xi}_f^F$, and χ are the shadow prices associated with these constraints and therefore also appear in Equation (2.1), which now reads

$$0 \leq -\lambda - \theta_f^{lim} \frac{d\Lambda(s)}{ds} q_f + \phi_f + \bar{\xi}_f^F - \underline{\xi}_f^F + \chi \perp q_f \geq 0 \quad \forall f \in \mathcal{F}. \quad (2.17)$$

By solving the augmented model comprising Equations (2.2), (2.16), and (2.17), with parameters $s^0 = s^{ref}$, λ^0 , η , and $\theta = \theta^{lim}$, we obtain a new equilibrium ($\lambda^* = \lambda^0, q^* =: q^{new}, s^* = s^0, \phi^* =: \phi^{new}$). Note that $\lambda^* = \lambda^0$ follows from the definition of the inverse demand function (2.3) and constraint (2.16c).

2.6. Definition of the calibration algorithm

From the previously introduced modules we can formulate a calibration algorithm for CV models.

- Step 1 (Initialization): Derive reference parameter values λ^{ref} , η^{ref} , and q^{ref} , as well as upper and lower bounds on these values $\underline{\lambda}^{ref}$, $\overline{\lambda}^{ref}$, $\underline{\eta}^{ref}$, and $\overline{\eta}^{ref}$, for instance from historical data. Set $s^0 \equiv s^{ref} = \sum_{f \in \mathcal{F}} q_f^{ref}$, set the iteration counter $i = 1$, and define $q^{ref,1} := q^{ref}$. Apply Module I (Section 2.3) to calculate $\phi^{ref,1}$ from $q^{ref,1}$.
- Step 2: Calculate $[\underline{\lambda}^{0,i}, \overline{\lambda}^{0,i}]$ and $[\underline{\eta}^i, \overline{\eta}^i]$ from $\phi^{ref,i}$ and $q^{ref,i}$ via Equations (2.13) and (2.14) (Module II).
- Step 3 (Termination): If $[\underline{\lambda}^{0,i}, \overline{\lambda}^{0,i}]$ and $[\underline{\eta}^i, \overline{\eta}^i]$ are non-empty and intersect with $[\underline{\lambda}^{ref}, \overline{\lambda}^{ref}]$ and $[\underline{\eta}^{ref}, \overline{\eta}^{ref}]$: Choose $\lambda^{0,i} \in [\underline{\lambda}^{0,i}, \overline{\lambda}^{0,i}] \cap [\underline{\lambda}^{ref}, \overline{\lambda}^{ref}]$, $\eta^i \in [\underline{\eta}^i, \overline{\eta}^i] \cap [\underline{\eta}^{ref}, \overline{\eta}^{ref}]$, calculate $\theta^i = \theta(\lambda^{0,i}, q^{ref,i}, s^{ref}, \phi^{ref,i}, \eta^i)$, and terminate the calibration. Otherwise, move to Step 4.
- Step 4 (Update): Apply Module III to obtain $q^{ref,i+1} := q^{new}$ and $\phi^{ref,i+1} := \phi^{new}$. Update $i = i + 1$, and move to Step 2.

Note that this algorithm is only guaranteed to terminate if the interval $[\underline{\lambda}^{ref}, \overline{\lambda}^{ref}]$ is chosen large enough with respect to all the reference values given. Unfortunately, “large enough” is difficult to quantify beforehand. As a consequence, the modeler might prefer to start with $\underline{\lambda}^{ref} = \overline{\lambda}^{ref} = \lambda^{ref}$, and gradually widen $[\underline{\lambda}^{ref}, \overline{\lambda}^{ref}]$ for those nodes and time periods stalling the algorithm. Furthermore, a less strict termination criterion can reduce the number of iterations; for instance $|s^* - s^{ref}| \leq TOL$ is suitable for most practical purposes, where s^* is the solution to the model comprising Equations (2.1) and (2.2), and TOL the maximum allowed deviation to the reference value.

3. Numerical example

We demonstrate the functionality of the proposed algorithm by applying it to the gas market model introduced by Baltensperger et al. [1]. The model represents the European Union (EU) markets and their main suppliers over 2 periods (summer and winter), consists of 43 nodes and 247 arcs, and is represented by 9432 complementarity conditions (including Equations (2.1) and (2.2)). The model equations and an exemplary model with two interconnected nodes are shown in Appendix C. The model and the calibration algorithm were implemented in MATLAB and solved by CPLEX. It takes a quad-core 3.4 GHz CPU 5.7 seconds on the average to compute one iteration of the algorithm, whereas the main computational burden originates from solving the augmented version of the model in Module III.

Historical data $(\lambda^{data}, q^{data}, s^{data}, \eta^{data})$ was obtained from various sources [10, 7, 8, 9, 13, 14, 15, 16, 18, 23]. As is often the case in practice, the total consumption did not match the reported sales of the traders $s^{data} \neq \sum_{f \in \mathcal{F}} q_f^{data}$.

A consistent set $(\lambda^{ref}, q^{ref}, s^{ref} \equiv \sum_{f \in \mathcal{F}} q_f^{ref}, \eta^{ref})$ was obtained by solving the

augmented version of the model comprising Equations (2.2), (2.16), and (2.17) before initializing the algorithm. We used $\lambda_{nt}^0 = \lambda_{nt}^{data}$, $\eta_{nt} = \eta_{nt}^{data}$, $\theta_{fnt} = 1$ for all $f \in \mathcal{F}, n \in \mathcal{N}, t \in \mathcal{T}$, where \mathcal{N} is the set of all nodes, and \mathcal{T} the set of all time periods. Equations (2.16a) and (2.16b) were altered to constrain q_{fnt} to the interval $[q_{fnt}^{data}, \infty)$, if $\sum_{f \in \mathcal{F}} q_{fnt}^{data} \leq s_{nt}^{data}$ and $\sum_{n \in \mathcal{N}} q_{fnt}^{data} \leq \frac{p_{ft}^{data}}{1 - LOSS_{ft}}$, where

p_{ft}^{data} is the total quantity produced by trader f in period t , and $LOSS_{ft}$ an estimate of the lost fraction of gas until it reaches the consumers. Otherwise, the q_{fnt}^{data} 's were scaled down to fit these inequalities, because s_{nt}^{data} and p_{ft}^{data} are among the most accurate figures available for the gas market. In other situations, different parameters might be prioritized. We obtain $(\lambda^{ref}, q^{ref}, s^{ref}, \phi^{ref}) = (\lambda^* = \lambda^{data}, q^*, s^* = s^{data}, \phi^*)$. For better readability, we drop subscripts n and t in the following.

In our example, we set $\underline{\lambda^{ref}} = \overline{\lambda^{ref}} = \lambda^{ref}$, $\underline{\eta^{ref}} = \max(-\eta^{ref} - 0.2, 0.3)$, and $\overline{\eta^{ref}} = \min(-\eta^{ref} + 0.2, 1)$. If $\lambda^{0,i}$ was set to $\underline{\lambda^{ref}}$ in iteration i , we decreased $\underline{\lambda^{ref}}$ by $0.02 \cdot \lambda^{ref}$ in iteration $i + 1$; we proceeded similarly when $\lambda^{0,i}$ hit the upper bound $\overline{\lambda^{ref}}$. The algorithm was set to terminate if neither $\underline{\lambda^{ref}}$ nor $\overline{\lambda^{ref}}$ were changed in an iteration, and $|s^* - s^{ref}| \leq 0.5$ million cubic meters per day (mcm/d).

For the chosen bounds, the algorithm terminates in the 10th iteration. The main characteristics of the solution are displayed in Table 3.1. Further details on the results are presented and discussed in Appendix B. The top half of Table 3.1 illustrates the deviations of the calibrated anchor values $\lambda^{0,10}$ and η^{10} to their reference values. These deviations increase with increasing misalignment between reference data and market equilibria the model can generate. Furthermore, $|q^{ref,10} - q^{ref}|$ increases with tighter bounds $[\underline{\lambda^{ref}}, \overline{\lambda^{ref}}]$ and $[\underline{\eta^{ref}}, \overline{\eta^{ref}}]$, and vice versa. Consequently, the modeler can distribute the deviations to

the parameters of his choice by setting $[\underline{\lambda}^{ref}, \overline{\lambda}^{ref}]$ and $[\underline{\eta}^{ref}, \overline{\eta}^{ref}]$ accordingly. Note that the infinite relative deviations in $q_f^{ref,10}$ and q_f originate from some $q_f^{ref,10} > 0$ and $q_f > 0$ while $q_f^{ref} = 0$. The low mean and median deviations indicate that the algorithm does not bias the results, for instance, towards a higher or lower average price level.

The lower half of Table 3.1 illustrates that the calibrated parameters $\lambda^{0,10}$, η^{10} , $\theta(\lambda^{0,10}, q_f^{ref,10}, s^0, \phi_f^{ref,10}, \eta^{10})$ (and $s^0 = s^{ref}$) indeed generate an equilibrium very close to the desired values $(\lambda^{0,10}, q_f^{ref,10}, s^0, \phi_f^{ref,10})$, which proves that the proposed algorithm works as intended when applied in practice.

Table 3.1: The upper rows display the deviations between parameters $\lambda^{0,10}$, $q_f^{ref,10}$, and $-\eta^{10}$, and their reference values. The lower rows show the deviations between the equilibrium values of the calibrated model and the calibrated parameters. Quantities are given in million cubic meters per day (mcm/d), prices in thousand Euros per million cubic meters (k€/mcm), and price elasticities are unitless.

Parameter /		Deviation between values				
variable		min max, abs.	min max, rel.	mean	median	
$\lambda^{0,10}$	λ^{ref}	-23.2 39.4 k€/mcm	-7.56 16.8 %	-0.34 k€/mcm	0.00 k€/mcm	
$-\eta^{10}$	$-\eta^{ref}$	-0.20 0.20	-37.4 59.0 %	-0.01	0.00	
$q_f^{ref,10}$	q_f^{ref}	-17.5 17.5 mcm/d	-100 ∞ %	0.00 mcm/d	0.00 mcm/d	
s^*	s^{ref}	-0.03 0.41 mcm/d	-0.03 0.25 %	0.00 mcm/d	0.00 mcm/d	
λ^*	$\lambda^{0,10}$	-3.15 0.26 k€/mcm	-0.83 0.07 %	-0.05 k€/mcm	0.05 k€/mcm	
q_f^*	$q_f^{ref,10}$	-0.07 0.05 mcm/d	-39.3 ∞ %	0.00 mcm/d	0.00 mcm/d	

Note that our algorithm reproduces the rudimentary algorithm introduced in Section 2.2 in the first iteration, if $[\underline{\lambda}^{ref}, \overline{\lambda}^{ref}] := [0, \infty)$ and $[\underline{\eta}^{ref}, \overline{\eta}^{ref}] := (-\infty, 0)$ are chosen. In this light, our algorithm is a significant extension, as it finds a solution with θ in the interval $[0, 1]$, and highlights the interplay between the parameters, which allows the user to take informed decisions on how the mismatch between model and reality is resolved.

4. Summary and Outlook

We propose an iterative algorithm to calibrate conjectural variations models for a network of markets. The algorithm builds upon three modules. In Module I, the marginal supply costs ϕ_f are derived for all traders f from the sales q_f^{ref} of the traders and the cost structure of the network. In Module II, all anchor prices $\lambda^0 \in [\underline{\lambda}^0, \overline{\lambda}^0]$ and price elasticities $\eta \in [\underline{\eta}, \overline{\eta}]$, which enable the equilibrium of the model to be in a certain point, are explored. In Module III, a new consistent and physically feasible set of sales q^{new} is obtained, which

changes the marginal costs of supply ϕ such that the ranges $[\underline{\lambda}^0, \overline{\lambda}^0]$ and $[\underline{\eta}, \overline{\eta}]$ are shifted in a desired direction.

The example indicates that the algorithm is able to calibrate a network of markets to real world data fast after a small number of iterations. The presented algorithm is broadly applicable, since the assumptions on which it is based are very mild and common in spatial partial equilibrium modeling. Furthermore, the algorithm can easily be configured to specific purposes, since the reference values and their tolerances can be chosen by the user. Finally, the algorithm can easily be expanded; for instance, one could follow the approach taken by Huppmann & Egging [12] and (manually) adjust the underlying marginal cost parameters of specific system services to provoke a more favorable outcome of the sales per trader.

Future work should emphasize on extending the update step of the algorithm, such that the *sizes* of the ranges of the various reference values can be weighed against each other and adjusted accordingly. In order to further increase the plausibility of the outcome with respect to economic theory, we also aim to improve how the algorithm sets these ranges relatively to each other. For instance, the market power parameters could be restricted to be larger in the high demand season than in the low demand season. Furthermore, we expect to achieve a reduction of iterations by exploiting the problem structure even further, particularly when determining the new set of sales per trader q^{new} in the update step.

Acknowledgements

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Appendix A. Calibration algorithm specific notation

Table A.1: Variables, parameters, and functions.

Abbr.	Explanation
q_f	Gas shipment of trader f
p_f	Total production of trader f
s	Total consumption
η	Price elasticity of demand
θ_f	Market power parameter of trader f
λ	Wholesale market price
Λ	Inverse demand function
ϕ_f	Marginal cost of trader f to supply quantity q_f
$\bar{\xi}_f^F$	Shadow price of enforcing an upper level on trader f 's sales q_f
$\underline{\xi}_f^F$	Shadow price of enforcing an lower level on on trader f 's sales q_f
χ	Shadow price of enforcing consumption level s^0

Table A.2: Sub- and superscripts.

Abbr.	Explanation
$(\cdot)_f$	Parameter/variable of trader f
$(\cdot)_n$	Parameter/variable in node n
$(\cdot)_t$	Parameter/variable in time period t
$(\cdot)^0$	Anchor value for inverse demand function
$(\cdot)^*$	Value of variable in equilibrium
$(\cdot)^{ref}$	Reference value for calibration
$(\cdot)^{data}$	Reference data, for instance based on historical values
$(\cdot)^{est}$	Estimated variable
$(\cdot)^{new}$	Updated variable
$(\cdot)^i$	Parameter/variable in iteration i
$\overline{(\cdot)}$	upper bound on parameter/variable
$\underline{(\cdot)}$	lower bound on parameter/variable

Table A.3: Models and sets.

Abbr.	Explanation
\mathcal{F}	Set of all traders
\mathcal{F}_+	Set of traders selling gas; $\mathcal{F}_+ = \{\mathcal{F} q_f > 0\}$
\mathcal{F}_0	Set of traders present in the market but not selling gas; $\mathcal{F}_0 = \{\mathcal{F} q_f = 0\}$
\mathcal{N}	Set of all nodes/markets
\mathcal{T}	Set of all time periods

Appendix B. In-depth analysis of results

As Figure B.1 depicts, the supplier's market shares after calibration $\frac{q_f^{10}}{s^{ref}}$ (lowermost bar in each country) are close to the reference values $\frac{q_f^{ref,1}}{s^{ref}}$ (topmost bar in each country) for most countries. We find the largest deviations between $\frac{q_f^{10}}{s^{ref}}$ and $\frac{q_f^{ref,1}}{s^{ref}}$ in countries with rather low overall consumption such as Bulgaria, the Czech Republic, Portugal, Slovakia, Slovenia, Sweden, and Switzerland; see Table B.1 for consumption values s^{ref} . At the same time, deviations are comparably low for the largest EU consumers Italy, Germany, the United Kingdom, France, and the Netherlands. We explain the dependency on the market size as follows: The calibration aims at finding a combination of ϕ^{ref} , λ^0 , and q^{ref} , which produces an equilibrium in itself. Therefore, the algorithm shifts the market shares $\frac{q_f^{ref}}{s^{ref}}$ for some f over the course of the iterations (Equation (2.15)). These impact the marginal supply costs ϕ^{ref} of all traders, but first and foremost of those traders f whose market shares $\frac{q_f^{ref}}{s^{ref}}$ are shifted. Once ϕ^{ref} leads to intersecting ranges $[\underline{\lambda}^0, \overline{\lambda}^0]$ and $[\underline{\lambda}^{ref}, \overline{\lambda}^{ref}]$, Equation (2.15) outputs $q^{est} = q^{ref}$, and $\frac{q_f^{ref}}{s^{ref}}$ stops shifting. Consequently, the largest shifts in market shares occur in those countries in which changes in $\frac{q_f^{ref}}{s^{ref}}$ have the lowest impact on the marginal costs ϕ^{ref} and therefore the range $[\underline{\lambda}^0, \overline{\lambda}^0]$, which are countries with low consumption compared to the total production of their supplier, the available pipeline capacity for imports, and the regasification capacity.

Furthermore, we observe that the changes from $\frac{q_f^{ref,i+1}}{s^{ref}}$ to $\frac{q_f^{ref,i}}{s^{ref}}$ are generally the largest in the first few iterations and decrease thereafter. Large changes indicate that the anchor price $\lambda^{0,i}$ changes, since this impacts the equilibrium heavily. This can be confirmed with help of Table B.1: for the example of Portugal in October-March, $\frac{\lambda^{0,10}}{\lambda^{ref}} = \frac{315 \text{ k€}/\text{mcm}}{275 \text{ k€}/\text{mcm}} \approx 1.15$, and as we increase $\overline{\lambda}^{ref}$ by 2% in the iteration $i+1$ if $\lambda^{0,i} = \overline{\lambda}^{ref}$ (Section 3), it is safe to assume that $\lambda^{0,i+1} = 1 + 0.02 \cdot i \cdot \lambda^{ref}$ for iterations $i \in \{1, \dots, 7\}$, and only stagnates thereafter. This is confirmed by the stagnating shares of Portugal after iteration $i = 7$ (Figure B.1).

We learn from Table B.1 that anchor prices $\lambda^{0,10}$ are overall slightly higher in the European winter than during summer. This intuitively makes sense, since consumption s^{ref} is clearly higher in winter than in summer, while production capacities remain unchanged, and therefore gas is scarcer. For the price elasticities η^{10} we do not observe such a clear trend, but note that the η^{10} remain close to reference values η^{ref} for the larger consumers United Kingdom, France, the Netherlands, and Spain. In Italy, $(-\eta^{10})$ is very low in both seasons. This is a consequence of the high wholesale prices λ^{ref} compared to the marginal supply costs ϕ^{ref} of the traders (Equation (2.10)). The algorithm concludes from this difference that consumers in Italy are willing to buy gas at high prices and therefore assigns low $(-\eta)$ and high market power parameters (Tables B.2 and B.3), which coincides with the interpretation an economist would make

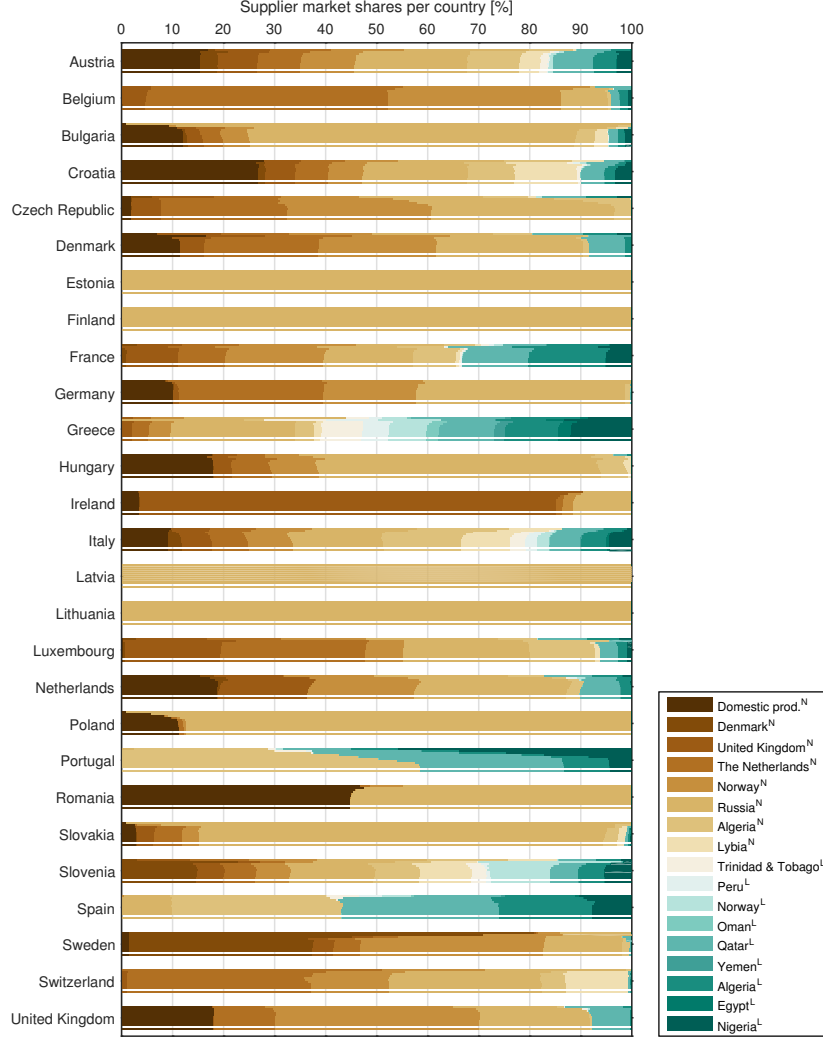


Figure B.1: Development of the market shares of all traders in all markets. For each country, a group of 10+1 bars is displayed, and each bar is divided into at most 17 sections. From top to bottom, the bars show the market shares $\frac{q_f^{ref,i}}{s^{ref}}$ of the suppliers in the iterations $i \in \{1, \dots, 10\}$. The lowermost bar represents the market shares in the calibrated equilibrium $\frac{q_f^{10}}{s^{ref}}$.

Table B.1: Reference consumption s^{ref} , reference price λ^{ref} , and reference price elasticity η^{ref} , as well as the calibrated anchor consumption s^0 , price $\lambda^{0,10}$ and price elasticity η^{10} . Quantities are given in million cubic meters per day (mcm/d), prices in thousand Euros per million cubic meters (k€/mcm), and price elasticities are unitless. $\lambda^{0,10}$ and η^{10} above their respective references are colored red with increasing absolute value, and below below their respective references blue with decreasing value, to visually underpin our findings.

	October-March					April-September				
	s^{ref}, s^0	λ^{ref}	$\lambda^{0,10}$	η^{ref}	η^{10}	s^{ref}, s^0	λ^{ref}	$\lambda^{0,10}$	η^{ref}	η^{10}
	[mcm/d]	[k€/mcm]	[k€/mcm]	[-]	[-]	[mcm/d]	[k€/mcm]	[k€/mcm]	[-]	[-]
Austria	34	354	349	-0.47	-0.35	16	354	354	-0.47	-0.30
Belgium	62	315	302	-0.44	-0.34	39	315	315	-0.44	-0.30
Bulgaria	10	413	392	-0.43	-0.30	6	413	389	-0.43	-0.30
Croatia	11	376	380	-0.44	-0.44	7	376	372	-0.44	-0.41
Czech Republic	34	291	313	-0.34	-0.34	13	291	291	-0.34	-0.54
Denmark	13	305	307	-0.40	-0.40	8	305	305	-0.40	-0.60
Estonia	3	381	381	-0.43	-0.63	2	381	381	-0.43	-0.63
Finland	14	381	373	-0.50	-0.50	9	381	373	-0.50	-0.50
France	189	333	333	-0.32	-0.32	71	333	331	-0.32	-0.30
Germany	287	305	305	-0.41	-0.61	161	305	301	-0.41	-0.30
Greece	14	383	382	-0.60	-0.42	12	383	365	-0.60	-0.47
Hungary	42	337	343	-0.35	-0.35	17	337	327	-0.35	-0.30
Ireland	15	298	306	-0.57	-0.58	13	298	295	-0.57	-0.38
Italy	286	376	375	-0.47	-0.31	161	376	376	-0.47	-0.30
Latvia	6	377	377	-0.35	-0.55	2	377	377	-0.35	-0.55
Lithuania	9	421	421	-0.53	-0.73	5	421	421	-0.53	-0.73
Luxembourg	4	315	314	-0.45	-0.30	3	315	314	-0.45	-0.45
The Netherlands	142	301	318	-0.46	-0.46	80	301	306	-0.46	-0.46
Poland	55	301	278	-0.37	-0.35	34	301	284	-0.37	-0.30
Portugal	14	275	315	-0.46	-0.46	14	275	302	-0.46	-0.46
Romania	42	177	207	-0.43	-0.43	31	177	177	-0.43	-0.63
Slovakia	21	345	333	-0.42	-0.30	8	345	333	-0.42	-0.30
Slovenia	3	376	368	-0.40	-0.38	2	376	354	-0.40	-0.32
Spain	102	294	316	-0.45	-0.45	85	294	303	-0.45	-0.45
Sweden	5	305	309	-0.53	-0.53	2	305	305	-0.53	-0.33
Switzerland	11	305	305	-0.30	-0.30	4	305	303	-0.30	-0.30
United Kingdom	256	298	301	-0.44	-0.44	158	298	299	-0.44	-0.44

when analyzing such a situation. In Germany, we face a special situation: from October to March, $(-\eta)$ is capped at its maximum value, while the opposite is true for April to September. As previously, this result originates from the given reference data. We do not judge at this point whether the outcome is realistic from an economic point of view; instead we emphasize that the algorithm allows the modeler to set the ranges $[\underline{\eta}^{ref}, \overline{\eta}^{ref}]$ such that the outcome is defensible.

Table B.2: Market power parameter values for all traders (T) in all markets (C) in October-March. The country abbreviations are given in Table B.4. Algeria and Norway supply the EU via the pipeline network and via liquefied natural gas (LNG) shipments. This is distinguished by N and L , respectively. Figures above 0.5 are colored red with increasing value, and below 0.5 green with decreasing value, to visually underpin our findings.

C \ T	AZ	DK	DZ ^L	DZ ^N	EG	GB	LY	NG	NL	NO ^L	NO ^N	OM	PE	QA	RU	TT	YE
AT	0.46	0.41	0.40	0.42	0.46	0.38	1	0.83	0.43	0	0.41	0.46	0.46	0.17	0.59	0.22	0.46
BE	0.09	0.09	0.09	0.09	0.09	0	0.09	0.09	0.01	0.09	0.03	0.09	0.09	0.09	1	0.09	0.09
BG	0.41	1	1	1	0.41	1	1	0.84	0.88	0.41	0.66	0.41	0.41	0.78	0.19	0.41	0.41
HR	0.69	1	0.83	0.82	0.69	0.55	0.36	0.35	0.64	0.69	0.76	0.69	0.69	0.25	0.83	0.69	0.69
CZ	0.14	0.14	0.14	0	0.14	0.14	0.14	0.14	0.03	0.14	0.05	0.14	0.14	0.14	0.28	0.14	0.14
DK	0.14	0.08	0.14	0.14	0.14	0.14	0.14	0.14	0	0.14	0.08	0.14	0.14	0.14	0.32	0.14	0.14
EE	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31
FI	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24
FR	0.27	0.19	0.11	0.12	0.27	0.21	0.27	0.26	0.28	0.27	0.40	0.27	0.27	0.11	0.54	0.98	0.27
DE	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.03	0.30	0.15	0.30	0.30	0.30	0.46	0.30	0.30
GR	0.62	0.62	0.61	0.66	1	0.80	0.62	0.44	0.68	0.54	0.65	0.65	0.71	0.73	0.62	0.60	1
HU	0.30	0.30	0.30	0.26	0.30	0.08	0	0.30	0.11	0.30	0.16	0.30	0.30	0.30	0.29	0.30	0.30
IE	0.22	0.22	0.22	0.22	0.22	0.09	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.93	0.22	0.22
IT	0.72	0.98	1	0.53	0.83	0.73	0.85	1	0.66	0.98	0.58	0	0.98	0.84	0.63	0.99	0.62
LV	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27
LT	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40
LU	0.13	0.13	0.13	0.13	0.13	0.03	0.13	0.13	0.03	0.13	0.24	0.13	0.13	0.13	0.33	0.13	0.13
NL	0.23	0.23	0.23	0.23	0.23	0.11	0.23	0.23	0.18	0.23	0.18	0.23	0.23	0	0.40	0.23	0.23
PL	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.11	0.20	0.20
PT	0.07	0.07	0.07	0.08	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0	0.07	0.07	0.07
RO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SK	0.22	0.22	0.22	1	0.22	0.60	0.75	0.22	0.37	0.22	1	0.22	0.22	0.22	0.13	0.22	0.22
SI	0.57	0.10	0.46	0.99	0.57	0.98	0.56	0.45	0.98	0.22	1	0.57	0	0.48	0.85	0.42	0.57
ES	0.09	0.09	0	0.21	0.09	0.09	0.09	0	0.09	0.09	0.09	0.09	0.09	0.01	0	0.09	0.09
SE	0.15	0	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.04	0.15	0.15	0.15	0.72	0.15	0.15
CH	0.15	0.15	0.15	1	0.15	0.15	0.24	0.15	0	0.15	0.05	0.15	0.15	0.15	0.26	0.15	0.15
GB	0.17	0.17	0.17	0.17	0.17	0.33	0.17	0.17	0	0.17	0.01	0.17	0.17	0.17	0.40	0.17	0.17

Tables B.2 and B.3 show that the market power parameters θ are similar per country and not per trader, for instance, Italy faces high θ , whereas Romania faces very low θ . This originates from the fact that the inverse demand function

Table B.3: Market power parameter values for all traders (T) in all markets (C) in April-September. The country abbreviations are given in Table B.4. Algeria and Norway supply the EU via the pipeline network and via LNG shipments. This is distinguished by N and L , respectively. Figures above 0.5 are colored red with increasing value, and below 0.5 green with decreasing value, to visually underpin our findings.

C \ T	AZ	DK	DZ ^L	DZ ^N	EG	GB	LY	NG	NL	NO ^L	NO ^N	OM	PE	QA	RU	TT	YE
AT	0.74	0.47	0.52	0.73	0.74	0.93	0.85	0.57	0.94	1	0.84	0.74	1	0.94	0.72	0.50	0.74
BE	0.34	1	0.40	1	0.34	0.47	0.34	0.60	0.16	0.34	0.15	0.34	0.34	0.56	1	0.34	0.34
BG	0.61	1	1	1	0.61	1	1	1	1	1	1	0.61	0.61	1	0.28	1	0.61
HR	0.94	0.66	0.81	1	0.94	0.99	0.98	0.69	1	0.94	0.99	0.94	0.94	0.94	0.96	0.25	0.94
CZ	0.34	0.34	0.34	0.14	0.34	0.11	0.34	0.34	0.15	0.34	0.33	0.34	0.34	0.34	0.79	0.34	0.34
DK	0.55	0.36	0.42	0.86	0.55	0.44	0.55	0.55	0.55	0.74	0.55	0.55	0.14	0.93	0.55	0.55	
EE	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37
FI	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28
FR	0.51	0.90	0.20	0.97	0.51	0.98	0.73	0.94	0.98	0.67	0.19	0.51	0.07	0.97	1	0.87	0.51
DE	0.32	0.32	0.32	0.57	0.32	1	0.32	0.32	0.18	0.32	0.20	0.32	0.32	1	0.26	0.32	0.32
GR	0.78	0.78	0.97	0.72	0.28	0.63	0.44	0.93	0.72	0.63	0.79	0	0.75	1	0.88	0.73	0.09
HU	0.38	0.38	0.38	1	0.38	0.82	1	0.38	1	0.38	1	0.38	0.38	0.38	0.19	0.38	0.38
IE	0.28	0.28	0.28	0.28	0.28	0.10	0.28	0.28	0.94	0.28	0.69	0.28	0.28	0.28	1	0.28	0.28
IT	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
LV	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31
LT	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45
LU	0.48	0.22	0.42	0.10	0.48	0.73	0.37	0.36	0.77	0.48	0.87	0.48	0.48	0.33	0.80	0.48	0.48
NL	0.42	0	0.26	0.16	0.42	0.33	0.42	0.40	0.44	0.42	0.43	0.42	0.42	0.23	0.63	0.42	0.42
PL	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.34	0.27	1	0.27	0.27	0.27	0.14	0.27	0.27
PT	0.14	0.14	0.07	0.30	0.14	0.14	0.14	0	0.14	0.14	0.14	0.14	0.14	0.06	0.14	0.14	0.14
RO	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
SK	0.48	1	1	1	0.48	0.80	1	1	1	0.48	1	0.48	0.48	1	0.22	0.48	0.48
SI	0.64	0.58	0.68	0.93	0.64	0.84	0.89	0.60	0.86	0.03	0.88	0.64	0	0.73	1	0.41	0.64
ES	0.17	0.17	0.07	0.50	0.17	0.17	0.17	0	0.17	0.17	0.17	0.17	0.17	0.09	0.11	0.17	0.17
SE	0.25	0.03	0.25	0.12	0.25	0.16	0.25	0.25	0.16	0.25	1	0.25	0.25	0.49	0.99	0.25	0.25
CH	0.37	0.37	0.88	1	0.37	0.62	0.28	0.37	0.11	0.37	0.40	0.37	0.37	0.94	0.46	0.37	0.37
GB	0.39	0.39	0.86	0	0.39	0.77	0.39	0.39	0.47	0.39	0.17	0.39	0.39	0.07	0.79	0.39	0.39

Table B.4: Country abbreviations. The attribution of a country into the “Eastern EU” or “Western EU” is determined by its geographical location and does not coincide with the historical political division of Europe.

Eastern EU		Western EU		Non-EU	
consumers		consumers		suppliers	
BG	Bulgaria	AT	Austria	AZ	Azerbaijan
CZ	Czech Republic	BE	Belgium	DZ ^L	Algeria (LNG)
EE	Estonia	CH	Switzerland	DZ ^N	Algeria (Pipeline)
FI	Finland	DE	Germany	EG	Egypt
GR	Greece	DK	Denmark	LY	Libya
HR	Croatia	ES	Spain	NG	Nigeria
HU	Hungary	FR	France	NO ^L	Norway (LNG)
LT	Lithuania	GB	United Kingdom	NO ^N	Norway (Pipeline)
LV	Latvia	IE	Ireland	OM	Oman
PL	Poland	IT	Italy	PE	Peru
RO	Romania	LU	Luxembourg	QA	Qatar
SI	Slovenia	NL	The Netherlands	RU	Russia
SK	Slovakia	PT	Portugal	TT	Trinidad & Tobago
		SE	Sweden	YE	Yemen

is equal for all traders, whereas the marginal supply costs ϕ vary per trader *and* country. The highest mean values were found for the large pipeline-bound suppliers Russia (0.50), Algeria^N (0.46), Norway^N (0.44), United Kingdom (0.43), Libya (0.41), and the Netherlands (0.40), which mirrors the observations we make in reality.

Moreover, the θ 's are generally *higher* in the European summer than in winter, which is rather counter-intuitive from an economic point of view; one would expect traders to exert less market power in times demand and prices are low. However, this finding can be explained by the interplay of the market power parameters θ and λ^0 . On one hand, Equation (2.4) implies that for non-zero reference sales q_f^{ref} the market power θ_f of a trader f increases with increasing difference between the market price and its marginal supply costs $\lambda^0 - \phi_f$. On the other hand, if $[\underline{\lambda}^0, \overline{\lambda}^0] \cup [\underline{\lambda}^{ref}, \overline{\lambda}^{ref}]$ contains more than one point, and q^{ref} , s^{ref} , ϕ^{ref} , and η^{ref} are given and fixed, the choice of λ^0 determines the θ_f , and any combination of $\lambda^0 \in [\underline{\lambda}^{ref}, \overline{\lambda}^{ref}]$ and $\theta_f(\lambda^0, q_f^{ref}, s^{ref}, \phi_f^{ref}, \eta^{ref}) \in [0, 1]$ is admissible. This renders a certain flexibility when choosing the parameters and the corresponding outcome. In our example, we chose reference prices λ^{ref} to be equal per country in the summer and winter periods. This corresponds to fixing a rather low $\lambda^0 \in [\underline{\lambda}^0, \overline{\lambda}^0] \cup [\underline{\lambda}^{ref}, \overline{\lambda}^{ref}]$ for the winter period, and a rather high $\lambda^0 \in [\underline{\lambda}^0, \overline{\lambda}^0] \cup [\underline{\lambda}^{ref}, \overline{\lambda}^{ref}]$ for the summer period. As a consequence, our calibration gives rather low θ 's in winter and rather high θ 's in summer. We carried out additional simulations, for which the results are not shown here, with a spread of 10 % and 20 % between summer and winter reference prices. For the former setting, the average θ is similar in both seasons, while in the latter the average θ is clearly higher in the high demand period. We conclude that the reference data greatly influences the obtained equilibrium and the corresponding parameters, and therefore should be carefully chosen.

Appendix C. Model equations and exemplary market setting

Appendix C is largely reproduced from Baltensperger et al. [1]. We introduce the model equations in Appendix C.1, show an exemplary market setting with two interconnected nodes in Appendix C.2, and introduce the associated notation in Appendix C.3. Note that we follow the convention used by Baltensperger et al. [1] and include producers in the notion of service providers, although producers are not an infrastructure service.

Appendix C.1. Model equations

Equations (C.1) describe the mechanics of the spatial partial equilibrium model of the European gas market in detail. We refrain from showing the loss terms in the equations to achieve a more compact notation.

$$0 \leq \quad LINC_{nt}^P + QUAC_{nt}^P q_{fnt}^P + \alpha_{nt}^P + \alpha_n^{PT} - \phi_{fnt}^N \perp q_{fnt}^P \geq 0 \quad \forall f, n, t \quad (\text{C.1a})$$

$$0 \leq \quad LINC_{nt}^I + \alpha_{nt}^I + \alpha_n^{IT} + \phi_{fnt}^N - \phi_{fn}^S \perp q_{fnt}^I \geq 0 \quad \forall f, n, t \quad (\text{C.1b})$$

$$0 \leq \quad LINC_{zt}^X + \alpha_{nt}^X + \alpha_n^{XT} - \phi_{fnt}^N + \phi_{fn}^S \perp q_{fnt}^X \geq 0 \quad \forall f, n, t \quad (\text{C.1c})$$

$$0 \leq \quad LINC_{nmt}^A + \alpha_{nmt}^A + \alpha_{nm}^{AT} - \phi_{fnt}^N + \phi_{fnt}^N \perp q_{fnt}^A \geq 0 \quad \forall f, n, m, t \quad (\text{C.1d})$$

$$0 \leq LINC_{nt}^L + \alpha_{nt}^L + \alpha_n^{LT} + LINC_{nmt}^B + \alpha_{nmt}^B + \alpha_{nm}^{BT} \\ + LINC_{mt}^R + \alpha_{mt}^R + \alpha_m^{RT} - \phi_{fnt}^N + \phi_{fnt}^N \perp q_{fnt}^B \geq 0 \quad \forall f, n, m, t \quad (\text{C.1e})$$

$$0 \leq \quad -\lambda_{nt}^C - \theta_{fnt}^C SLP_{nt}^C q_{fnt}^C + \phi_{fnt}^N \perp q_{fnt}^C \geq 0 \quad \forall f, n, t \quad (\text{C.1f})$$

$$0 \leq \quad q_{fnt}^P + q_{fnt}^X + \sum_{m \in \mathcal{A}(n)} q_{fnt}^A + \sum_{m \in \mathcal{B}(n)} q_{fnt}^B \\ - q_{fnt}^I - q_{fnt}^C - \sum_{m \in \mathcal{A}(n)} q_{fnt}^A - \sum_{m \in \mathcal{B}(n)} q_{fnt}^B \perp \phi_{fnt}^N \geq 0 \quad \forall f, n, t \quad (\text{C.1g})$$

$$0 \leq \quad \sum_{t \in \mathcal{T}} q_{fnt}^I - \sum_{t \in \mathcal{T}} q_{fnt}^X \perp \phi_{fn}^S \geq 0 \quad \forall f, n \quad (\text{C.1h})$$

$$0 \leq \quad \overline{CAP}_{zt}^Z - \sum_{f \in \mathcal{F}(z)} q_{fzt}^Z \perp \alpha_{zt}^Z \geq 0 \quad \forall z, t \quad (\text{C.1i})$$

$$0 \leq \quad \overline{CAP}_z^{ZT} - \sum_{t \in \mathcal{T}} \sum_{f \in \mathcal{F}(z)} q_{fzt}^Z \perp \alpha_z^{ZT} \geq 0 \quad \forall z \quad (\text{C.1j})$$

$$0 \leq \lambda_{nt}^C - \left(INT_{nt}^C + SLP_{nt}^C \sum_{f \in \mathcal{F}(n)} q_{fnt}^C \right) \perp \lambda_{nt}^C \geq 0 \quad \forall n, t \quad (\text{C.1k})$$

Appendix C.2. Graphical illustration

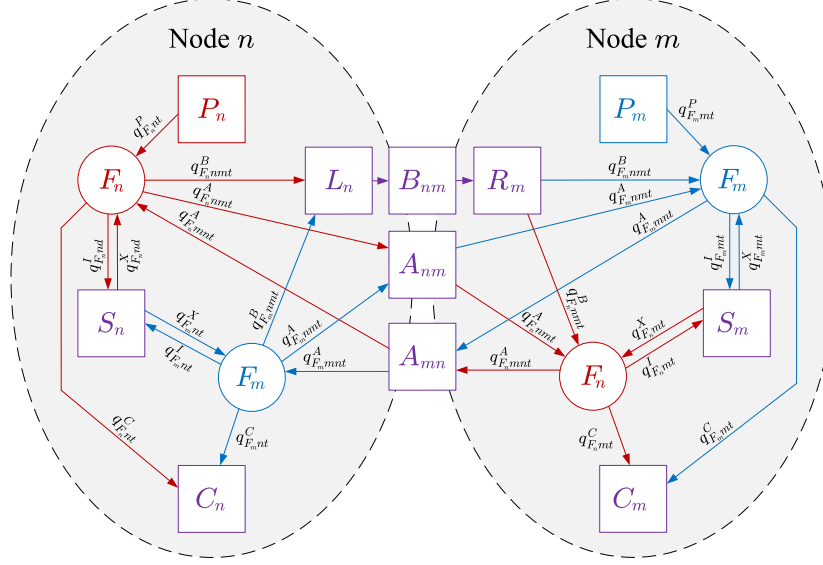


Figure C.1: Gas market model with two nodes. P : producer. A : pipeline operator. L : liquefaction plant operator. B : LNG shipment. R : regasification plant operator. S : storage operator. C : consumer. F_n : trader associated with producer P_n . F_m : trader associated with producer P_m . q_{fnt}^P : quantity delivered from producer to trader f at node n in time period t . q_{fnt}^A : pipeline transportation of trader f via arc nm in period t . q_{fnt}^B : LNG shipment of trader f via arc nm in period t . q_{fnt}^I : storage injection by trader f at node n in period t . q_{fnt}^X : storage extraction by trader f at node n in period t . q_{fnt}^C : sales by trader f to consumer in node n in period t . The traders F_n and F_m , their decision variables, and their corresponding producers P_n and P_m are colored red (n) and blue (m), respectively. Service providers, except producers, and flows between them are marked purple, as well as the consumers, since all traders trade with them.

Appendix C.3. Notation

Table C.1: This table introduces the nomenclature concerning service providers, traders and consumers.

Service providers, traders and consumers	
A_{nm}	Transmission system operator of pipeline nm
B_{nm}	Shipping company transporting LNG from n to m
C_n	Consumer at node n
I_n	Storage operator injecting gas at node n
L_n	Liquefaction plant operator at node n
P_n	Gas producing company at node n
R_n	Regasification plant operator at node n
S_n	Storage operator at node n
F_n	The trader associated with producer at node n
X_n	Storage operator extracting gas at node n
Z_z	Placeholder for a service provider ($P_n, I_n, X_n, L_n, R_n, A_{nm}, B_{nm}$) at node n / arc nm

Table C.2: This table introduces all sets used for the mathematical description of the model.

Sets	
$t \in \mathcal{T} = \{T_1, \dots, T_{\bar{t}}\}$	A time period t in the set \mathcal{T} of all periods of a year
$n, m \in \mathcal{N} = \{N_1, \dots, N_{\bar{n}}\}$	Nodes n, m in the set \mathcal{N} of all nodes
$f \in \mathcal{F} = \{F_1, \dots, F_{\bar{n}}\}$	A trader f in the set \mathcal{F} of all traders
$z \in \mathcal{Z}$	A node/arc element from the set \mathcal{Z}
$\mathcal{A} \subset \mathcal{N} \times \mathcal{N}$	Set of arcs connecting 2 nodes by pipeline
$\mathcal{B} \subset \mathcal{N} \times \mathcal{N}$	Set of arcs connecting 2 nodes by ship
$\mathcal{C} \subseteq \mathcal{N}$	Set of nodes at which a consumer is active
$\mathcal{I} \subseteq \mathcal{N}$	Set of nodes at which storage injection is possible
$\mathcal{L} \subseteq \mathcal{N}$	Set of nodes at which a liquefaction terminal operator is active

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Table C.2 – *Continued from previous page*

Sets	
$\mathcal{P} \subseteq \mathcal{N}$	Set of nodes at which a gas producer is active
$\mathcal{R} \subseteq \mathcal{N}$	Set of nodes at which a regasification terminal operator is active
$\mathcal{X} \subseteq \mathcal{N}$	Set of nodes at which storage extraction is possible
$\mathcal{Z} \in \{\mathcal{P}, \mathcal{L}, \mathcal{B}, \mathcal{R}, \mathcal{A}, \mathcal{I}, \mathcal{X}\}$	Placeholder for the set of nodes/arcs at which a type of service provider is active
$\mathcal{A}(n) \subseteq \mathcal{N} \setminus \{n\}$	Set of nodes which are connected to n by pipeline
$\mathcal{B}(n) \subseteq \mathcal{N} \setminus \{n\}$	Set of nodes which are connected to n by ship
$\mathcal{C}(f) \subseteq \mathcal{N}$	The set of all nodes with consumers which are reachable by trader f
$\mathcal{N}(f) \subseteq \mathcal{N}$	The set of all nodes which are reachable by trader f
$\mathcal{F}(z)$	The set of all traders active at node/arc z
$\mathcal{Z}(f)$	The set of all nodes/arcs in which service Z is active and are reachable by trader f

Table C.3: The *parameters* are generally described by capital Roman letters. Lower-case Roman letters are only chosen if the parameter is directly linked to a variable of the same name. Occasionally, lower-case Greek letters are chosen to follow conventions. The superscripts indicate whether the parameter is related to a service provider of type $Z \in \{P, L, B, R, A, I, X\}$ or a consumer C . Subscripts indicate the trader f , node/arc z , and the period of the year t the parameter is related to.

Parameters	
\overline{CAP}_{nt}^Z	Maximum capacity of service Z located at z in period t
\overline{CAP}_n^{ZT}	Maximum capacity of service Z located at z over all periods \mathcal{T}
INT_{nt}^C	Maximum willingness to pay (intercept of inverse demand function with the $s_{nt}^C = 0$ - axis) of consumers at node n in period t
$LINC_{zt}^Z$	Linear cost function term for service Z located at z in period t
$LOSS_z^Z$	Loss factor when using service Z located at z
$QUAC_{zt}^Z$	Quadratic cost function term for service Z located at z in period t
SLP_{nt}^C	Slope of the inverse demand curve of the consumers at node n in period t , is assumed strictly negative

Continued on next page

Table C.3 – *Continued from previous page*

Parameters	
θ_{fnt}	Market power parameter of trader f in node n and period t

Table C.4: The *variables* are described by lowercase letters. Primal variables are Roman, while dual variables are Greek letters. The superscripts indicate whether the variable is related to a service provider of type $Z \in \{P, L, B, R, A, I, X\}$, a consumer C , or a node N . Subscripts indicate the trader f the variable corresponds to, at which node/arc z the transaction or service is located, and in which period of the year t it takes place.

Variables	
q_{fnt}^C	Flow of trader f to consumer C at node n in period t
q_{fzt}^Z	Flow between trader f and service provider Z at node/arc z in period t
α_{zt}^Z	Congestion fee of service Z located at z in period t
α_z^{ZT}	Congestion fee on annual usage of service Z located at z
ϕ_{fnt}^N	Dual variable of the volume balance of trader f at node n and period t
ϕ_{fn}^S	Dual variable of the annual volume balance of trader f in storage S at node n
λ_{nt}^C	Wholesale price at node n in period t

Table C.5: This table introduces the *functions*. The superscript C indicates that the function is related to the wholesale market. Subscripts indicate at which node n and in which period of the year t the function is valid.

Functions	
$\Lambda_{nt}^C(s_{nt}^C)$	Inverse demand function of consumer C at node n in period t .

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